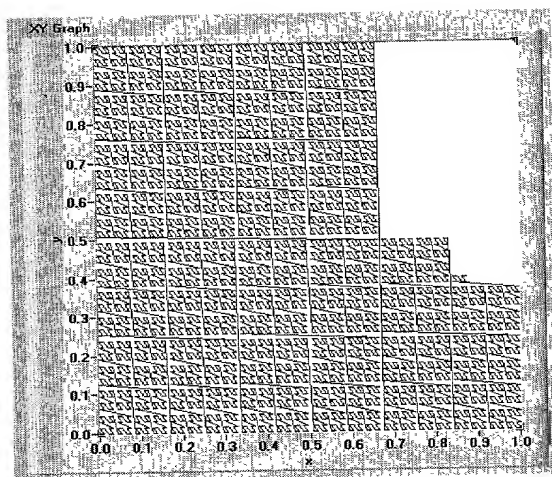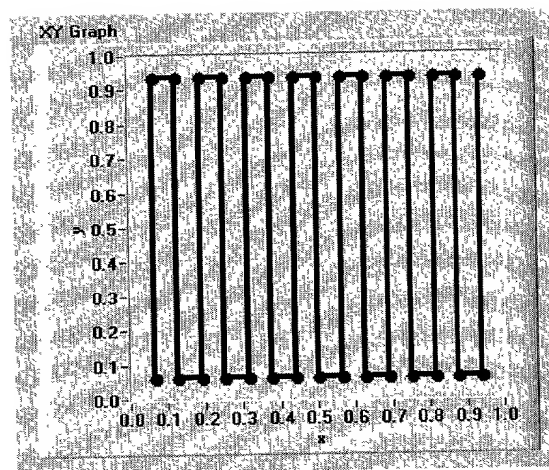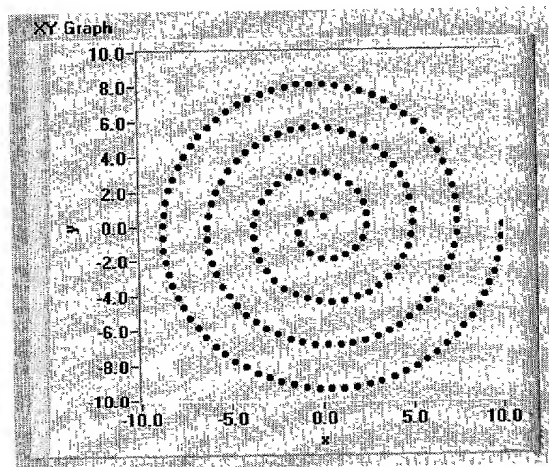Approximated Peano Curve. The
space-filling process has not been
completed.

## Figure 1A (Prior Art)



Boustrophedon Path

## Figure 1B (Prior Art)



Archimedes Spiral defined by equally
distributed points

## Figure 1C (Prior Art)
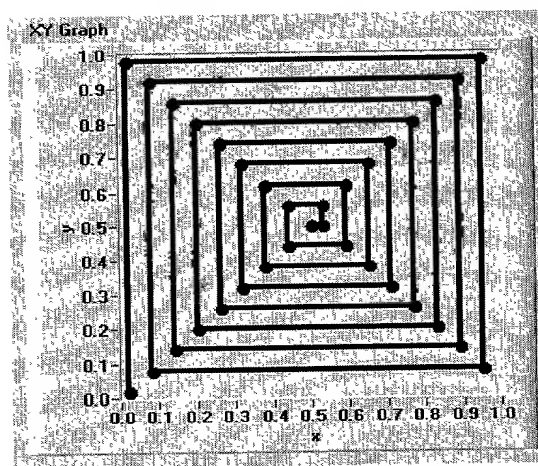


Spiral-like line-based scanning

## Figure 1D (Prior Art)

camera
110

120

125

Host Computer System
102

Figure 2A

Computer
System
102

I/O
206

Memory
204

Processor
208

Figure 2B

camera 110C

110D

110B

camera 110A

140

computer
system 102A

manufacturing
apparatus
106

objects 120

REJECTED

rejected
object 122

Figure 3A

robotic
arm
114B

camera
110F

120

camera
110E

robotic
arm
114A
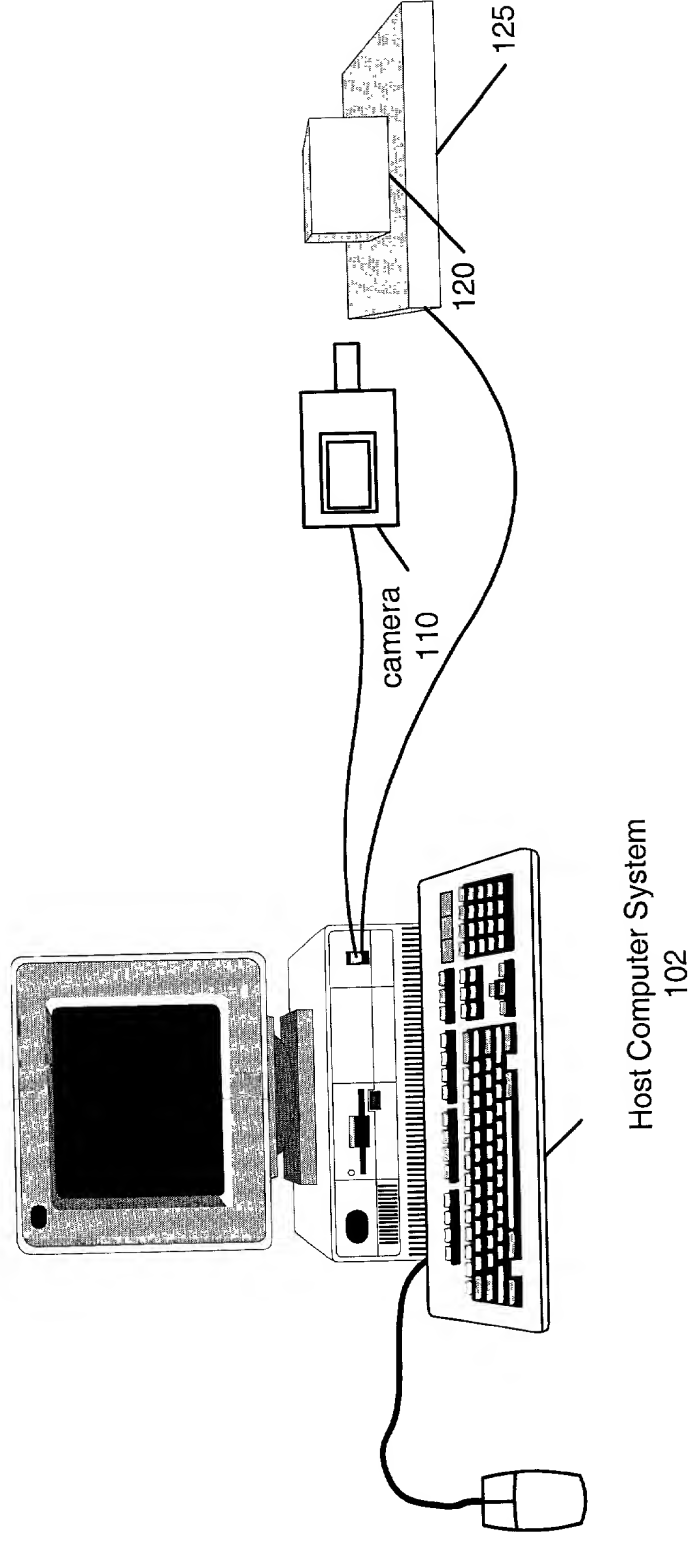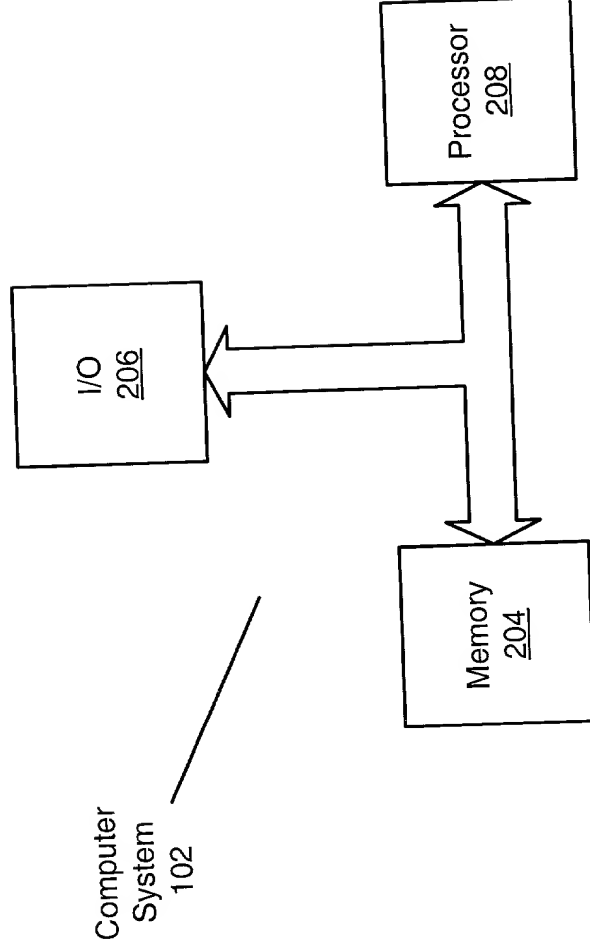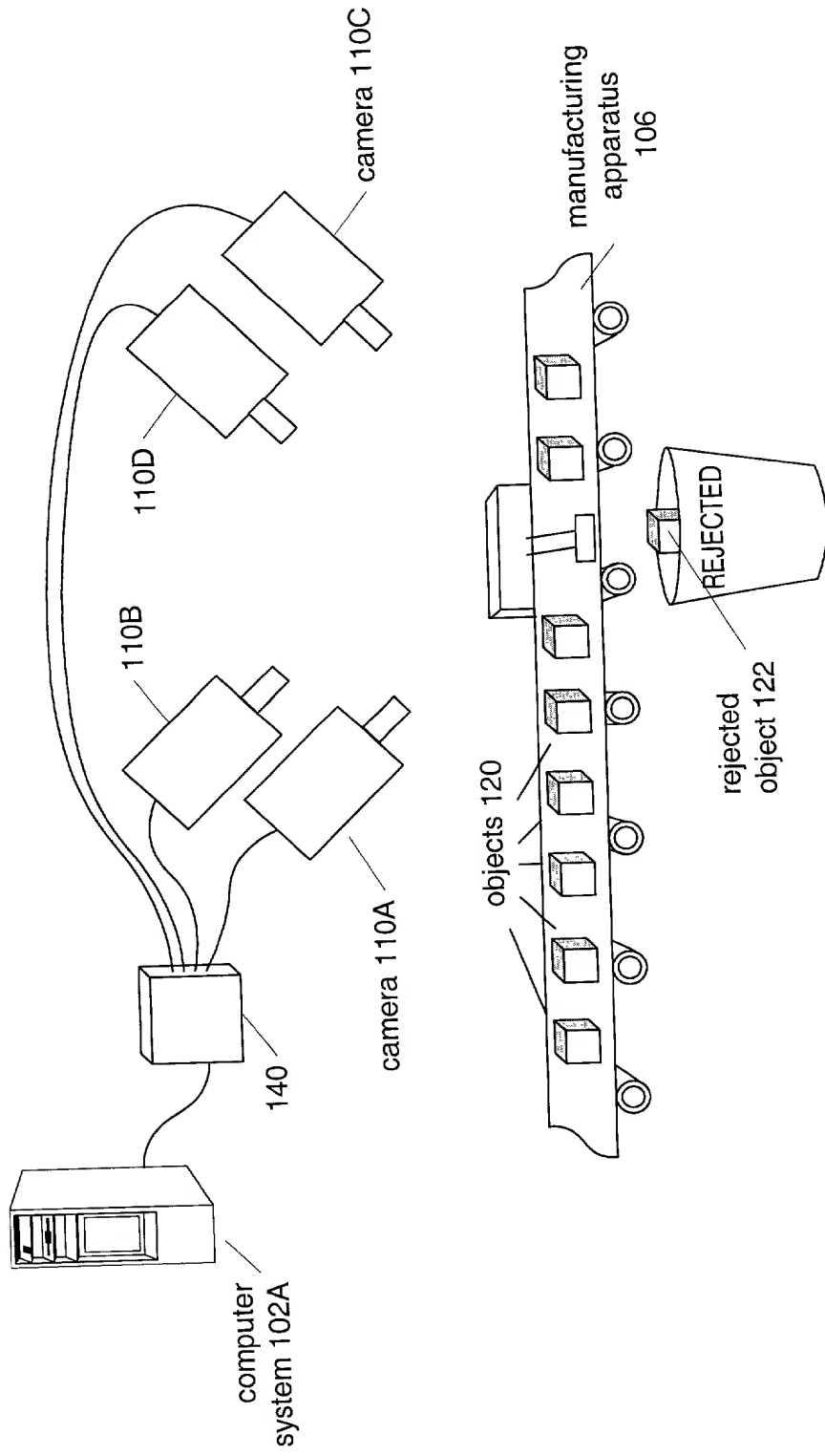
computer
system 102B
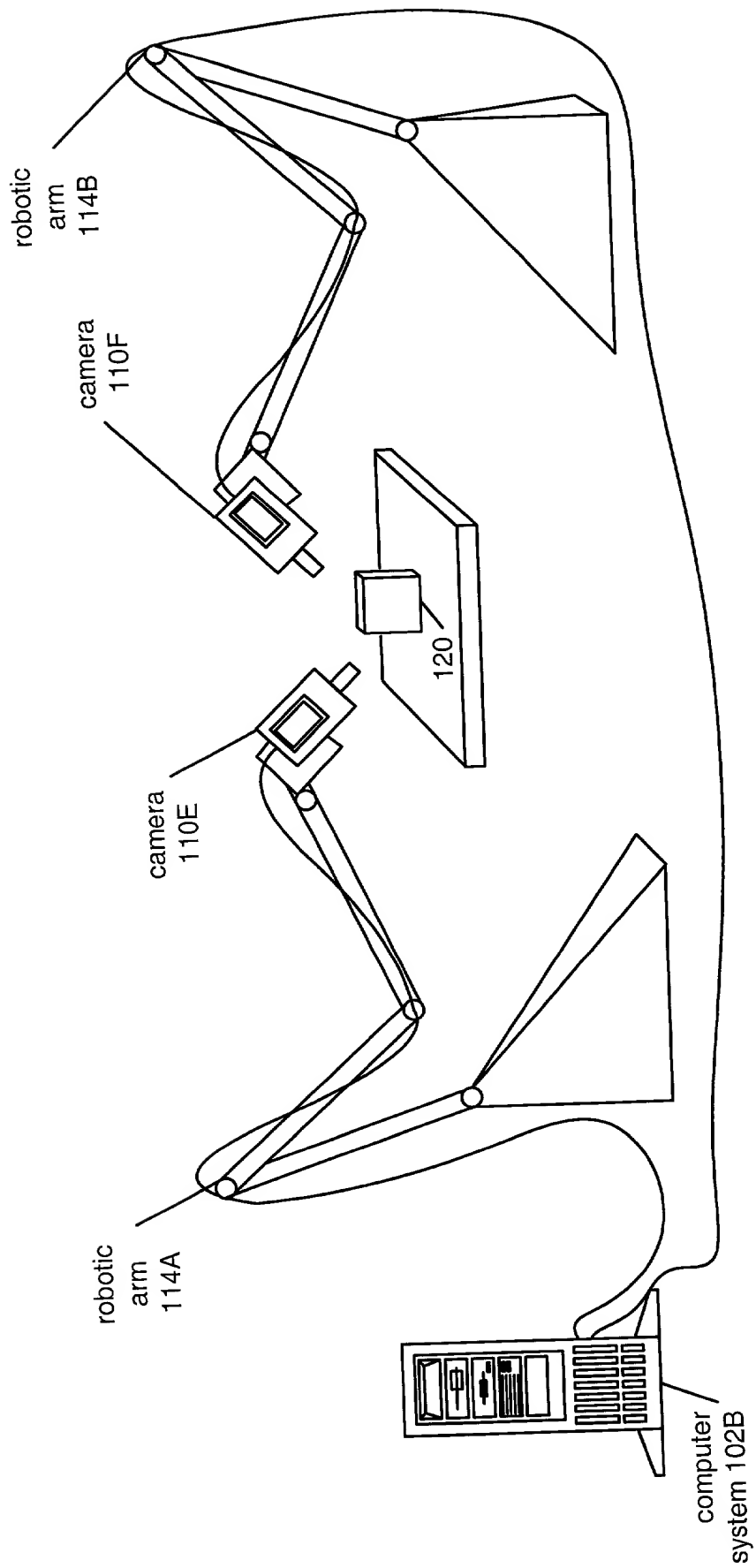
Figure 3B

Computer System
102C

Array Elements
304

Phased Array
306

Figure 3C

Optical Fiber
320A

Laser Beam
312

Optical Fiber
320B

Motion Stage
330

Laser
310

Computer System
102D
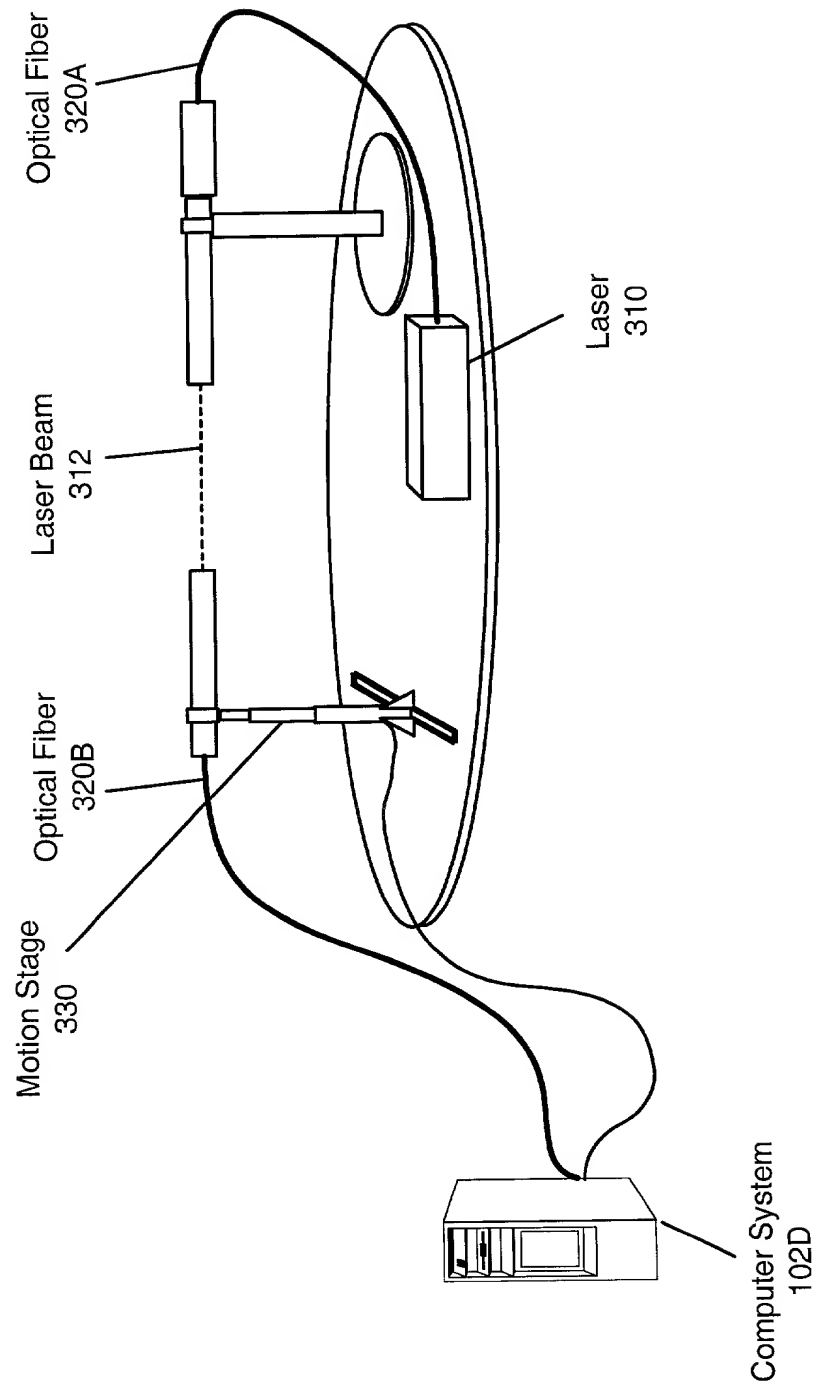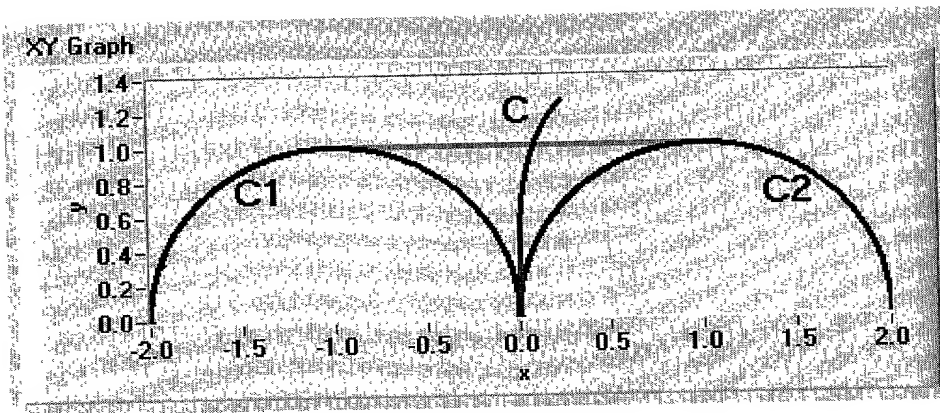
Figure 3D

The situation of Lemma 1

Figure 4A
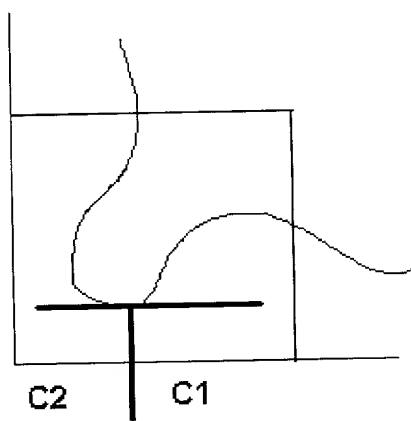


Case (A)

Figure 4B
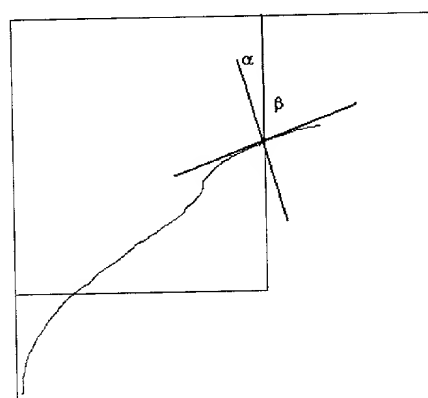


Case (B)

Figure 4C
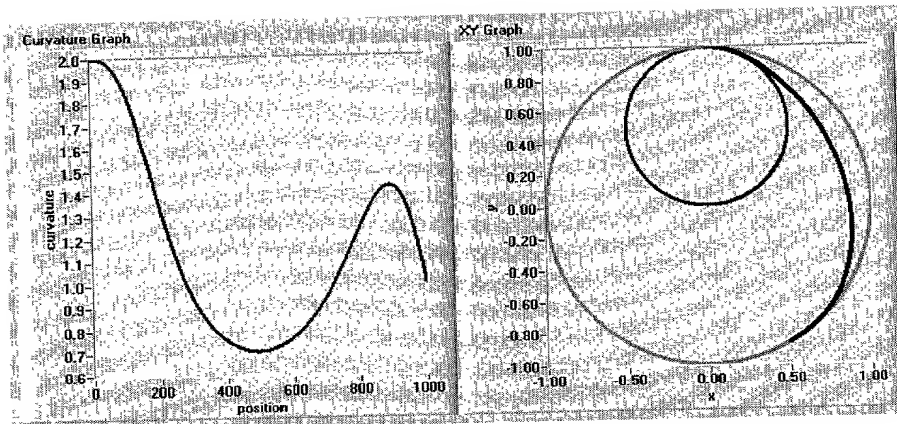
Smooth transition between two circles of different radii.

Figure 4D
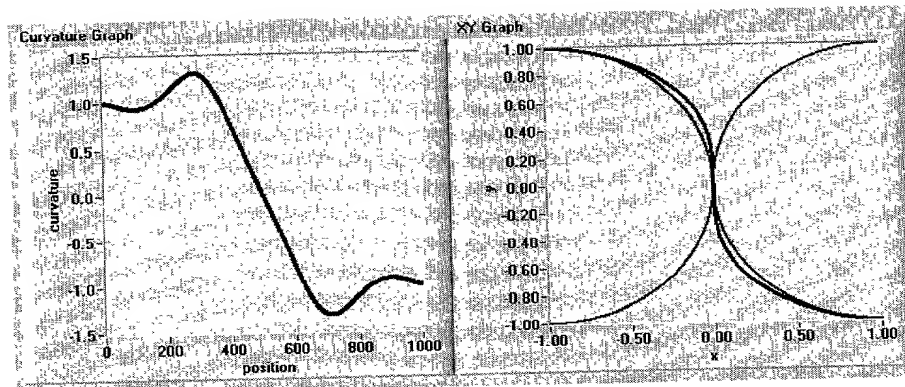
Smooth transition between two circles of same radius.

Figure 4E
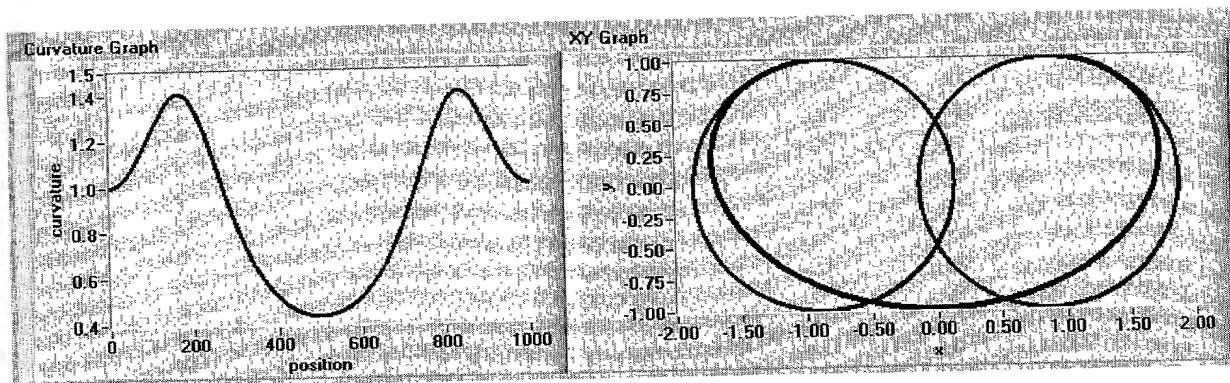
Transition between two unit circles of radius 1. The distance
between the circles is sqrt(3)
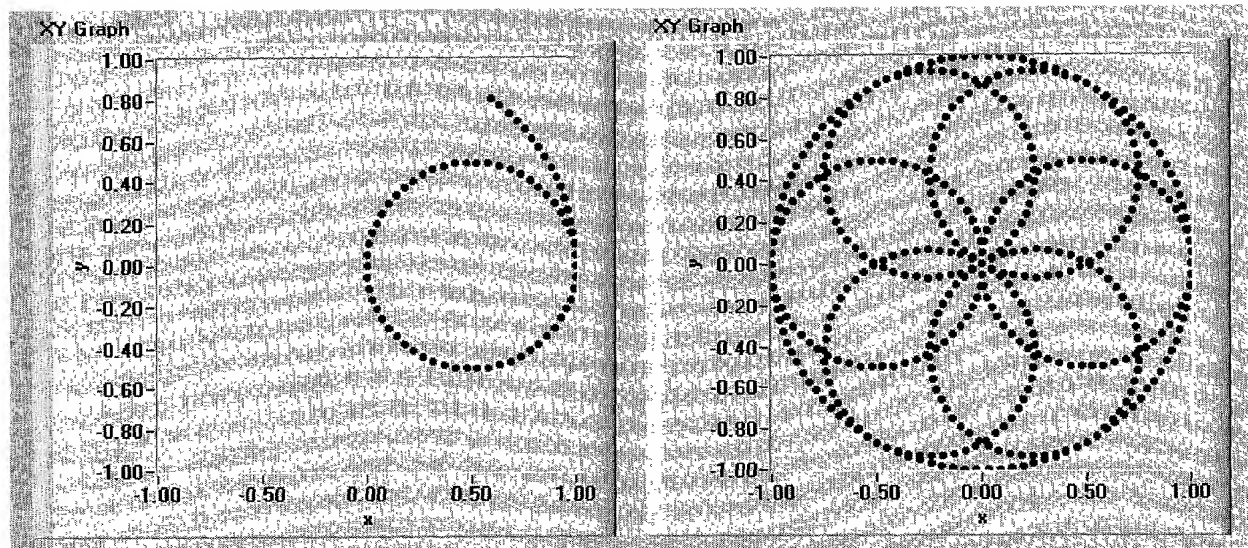
Figure 4F

Beginning (left) and completion (right) of a scanning scheme where the curvature
is below a certain value

## Figure 5A
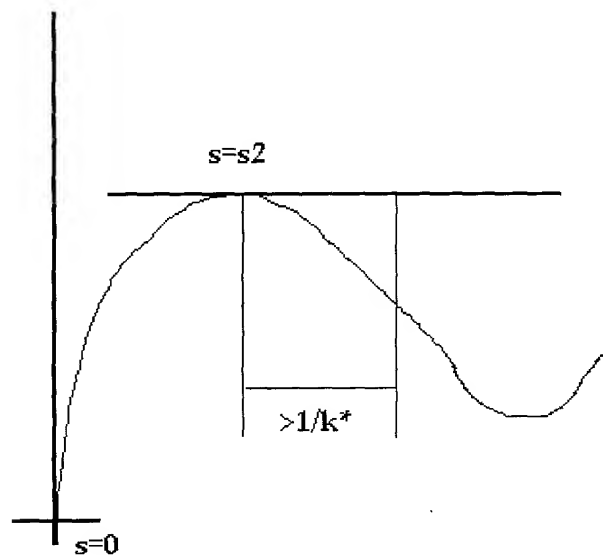


Construction of s2 and the subsequent part of the curve

## Figure 5B
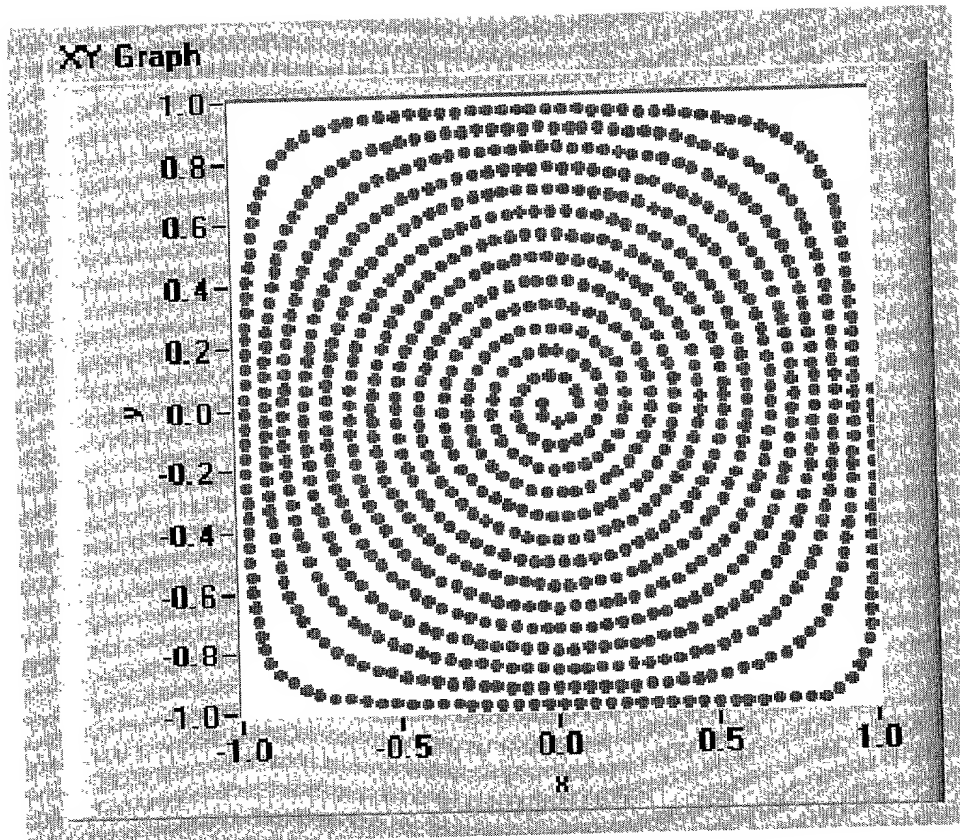
Conformal Spiral.

Figure 6

```
                    ┌─────────┐
                    │  Start  │
                    └────┬────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │     Place Object In Scan Region     │
        │                702                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │ Determine Characteristic Geometry of Scan Region │
        │                704                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │    Generate Conformal Scan Curve Based on    │
        │         Characteristic Geometry      │
        │                706                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │  Scan Region Using the Conformal Scan Curve  │
        │                708                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │ Determine One or More Characteristics of the Object │
        │                710                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
        ┌────────────────────────────────────┐
        │ Generate Output Indicating the Determined Object │
        │              Characteristics         │
        │                712                  │
        └────────────────┬───────────────────┘
                         │
                         ▼
                    ┌─────────┐
                    │   End   │
                    └─────────┘
```
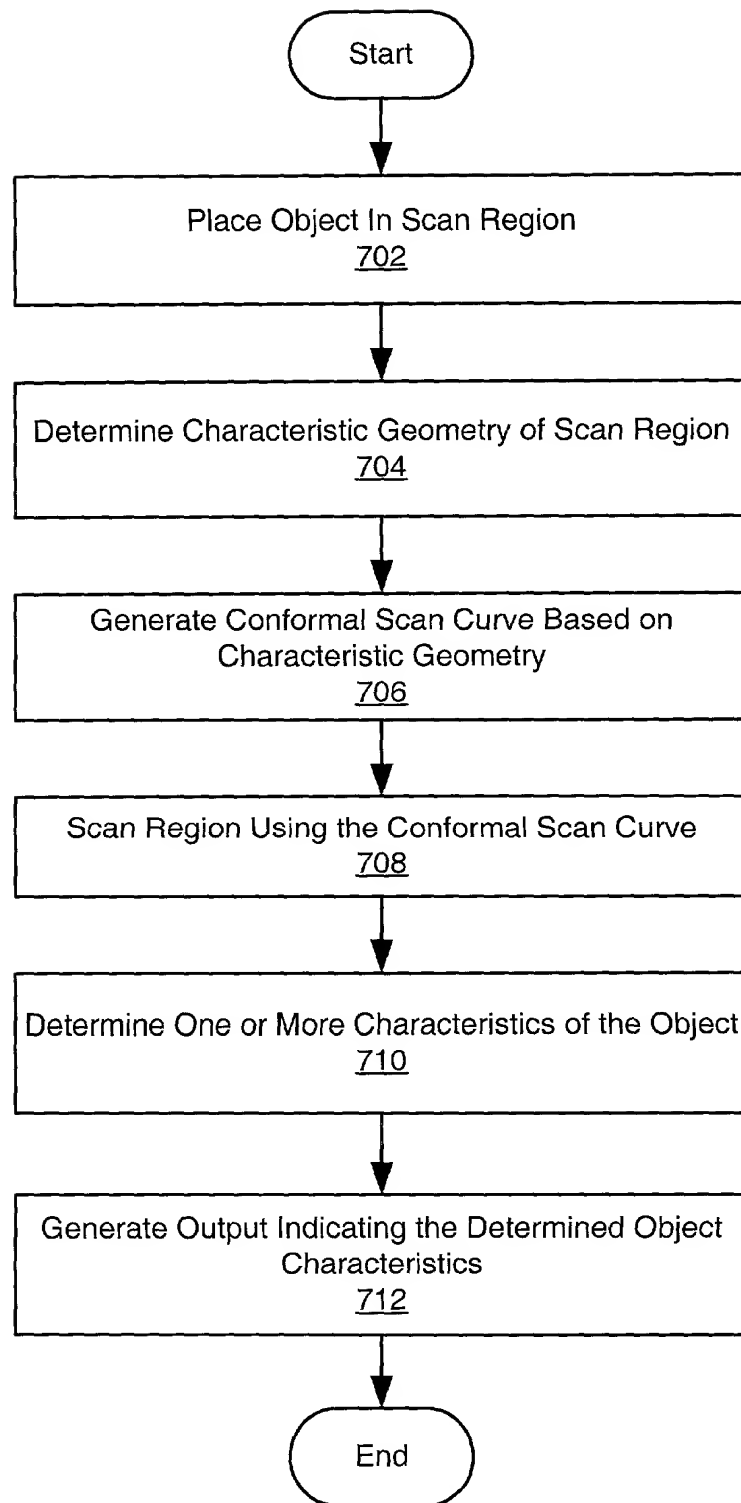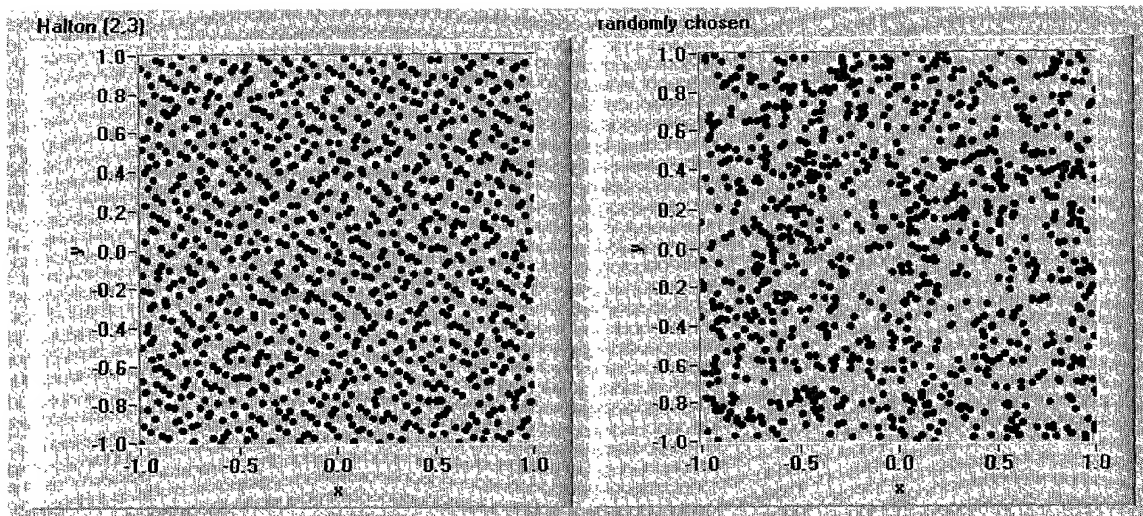
Figure 07

The first 1000 Halton points (left) and randomly chosen points (right)

Figure 8A
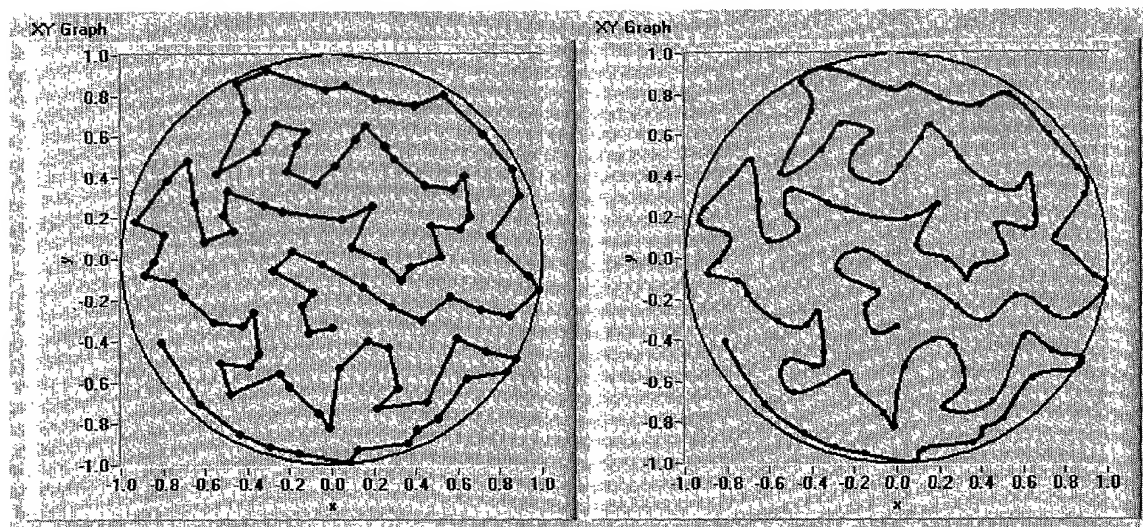


Original solution (left) and splined version (right).

Figure 8B

Start

Calculate Low Discrepancy Sequence for Scan
Region
902

Calculate a Motion Control Trajectory Based On the
Low Discrepancy Sequence
904

Scan Region Using Calculated Motion Control
Trajectory
906

Determine Object Characteristics
908

Generate Output Indicating the Determined Object
Characteristics
910

End

Figure 9

Definition of $I_1$, $I_2$, and $I_3$

Figure 10
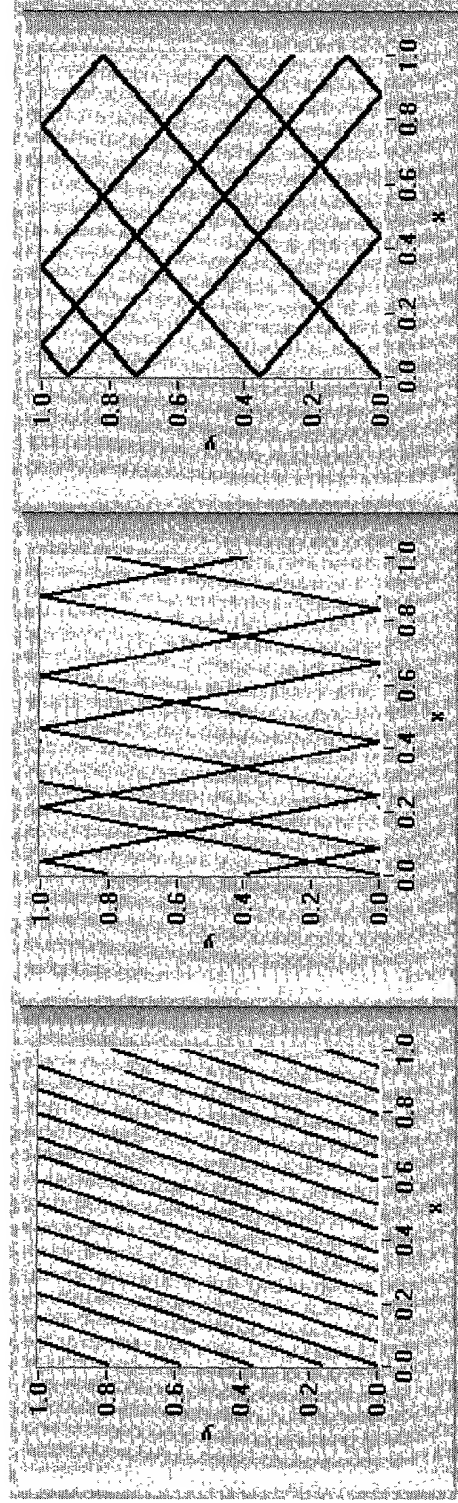
Figure 11A

Figure 11B

Figure 11C

Generate Unbounded Low Discrepancy Point
1222

Apply Boundary Conditions to the Unbounded Low
Discrepancy Point to Generate a Bounded Low
Discrepancy Point in a Region
1224

Done?
1226

No

Yes

Store Generated Low Discrepancy Sequence of
Points Representing a Low Discrepancy Curve in the
Region
1228

Output Generated Low Discrepancy Sequence of
Points Representing the Low Discrepancy Curve
1230

Figure 12A

Select a Pair of Irrational Numbers (alpha1,alpha2)
such that the Sequence ({(n*alpha1)mod1},
{(n*alpha2) mod1}) for all Natural Numbers n is a
LDS in the Unit Square.
1202

Select a Length, L, and a Step Rate, epsilon, of the
LD Curve in the Unit Square
1204

Initialize Current Length, l, to Zero, and Initialize
Current Position (x ,y), (e.g., to (0,0)
1206

Increment x and y and Apply Boundary Conditions at
Borders of Unit Square (e.g., Toroidal, Reflectance,
or Both), Generating a Low Discrepancy Sequence
Point $(x_n, y_n)$
1208

l < L?
1210

No

Yes

Output Generated Low Discrepancy Sequence of
Points Representing the Low Discrepancy Curve
1212

Figure 12B

Beginning of a Low Discrepancy Curve based on a specific
Halton Sequency in 2d
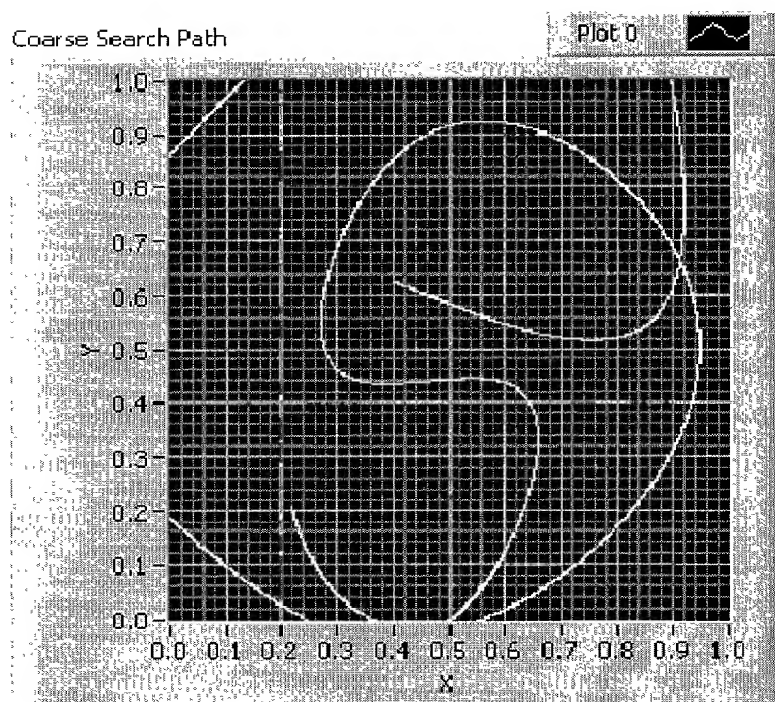
## Figure 13A



Splined Low Discrepancy Curve coarse search

## Figure 13B
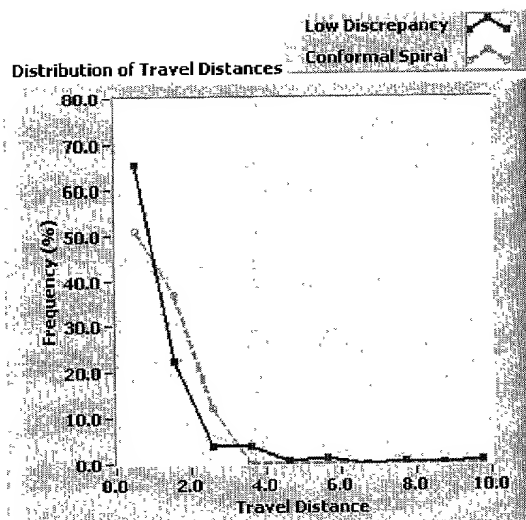
Comparison of Conformal Spiral and Low Discrepancy Searching

# Figure 13C
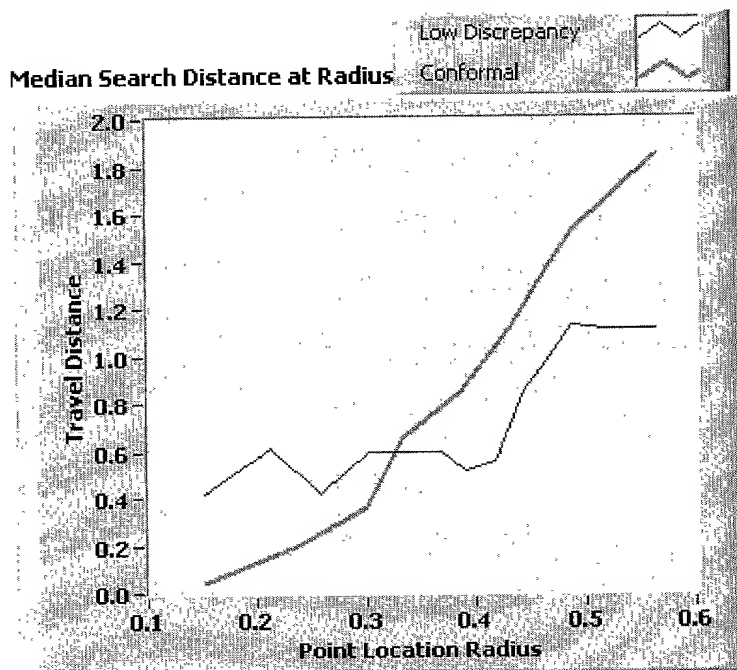


Comparison of Travel Distance for Low Discrepancy Search and
Conformal Spiral Search

# Figure 13D

```
        ┌─────────┐
        │  Start  │
        └─────────┘
             │
             ▼
┌─────────────────────────────────────────┐
│   Calculate Low Discrepancy Sequence     │
│            for Scan Region                │
│                 1402                      │
└─────────────────────────────────────────┘
             │
             ▼
┌─────────────────────────────────────────┐
│   Calculate a Low Discrepancy Curve       │
│       Based On the Low Discrepancy        │
│               Sequence                    │
│                 1404                      │
└─────────────────────────────────────────┘
             │
             ▼
┌─────────────────────────────────────────┐
│   Scan Region Using Calculated Low        │
│          Discrepancy Curve                │
│                 1406                      │
└─────────────────────────────────────────┘
             │
             ▼
┌─────────────────────────────────────────┐
│       Determine Object Characteristics    │
│                 1408                      │
└─────────────────────────────────────────┘
             │
             ▼
┌─────────────────────────────────────────┐
│   Generate Output Indicating the          │
│     Determined Object Characteristics     │
│                 1410                      │
└─────────────────────────────────────────┘
             │
             ▼
        ┌─────────┐
        │   End   │
        └─────────┘
```
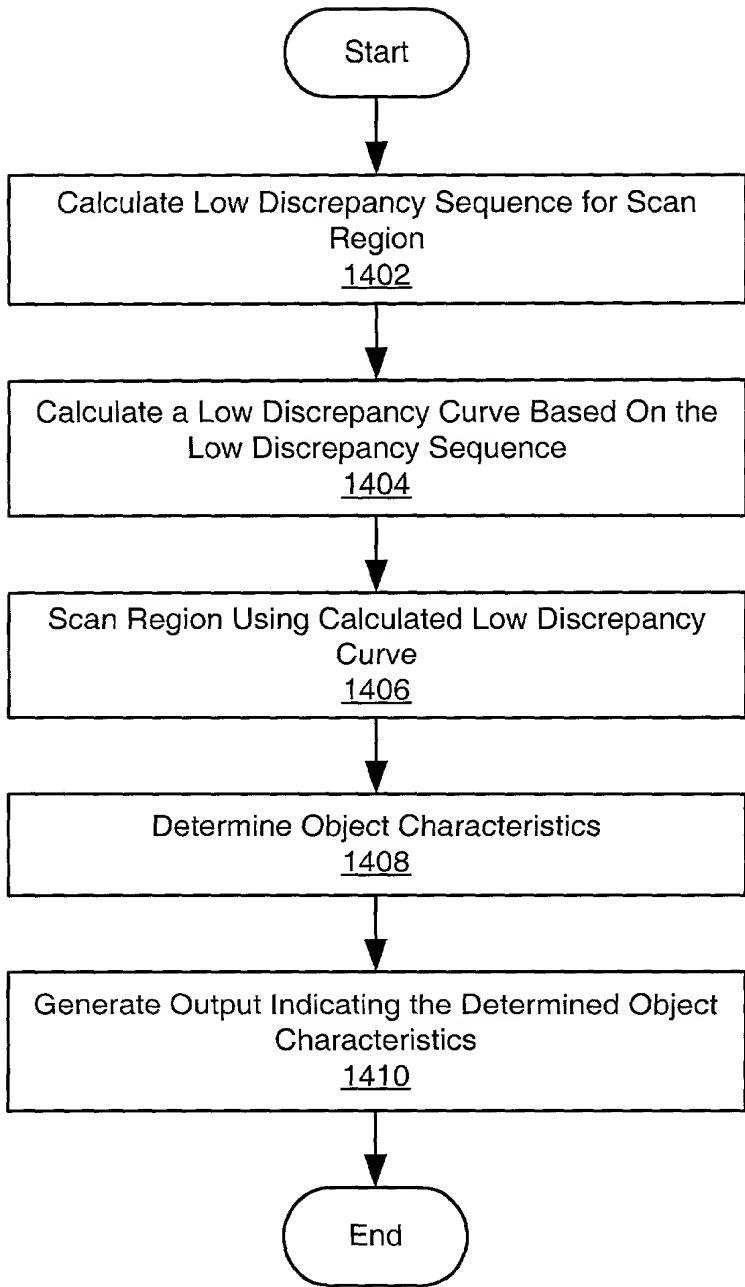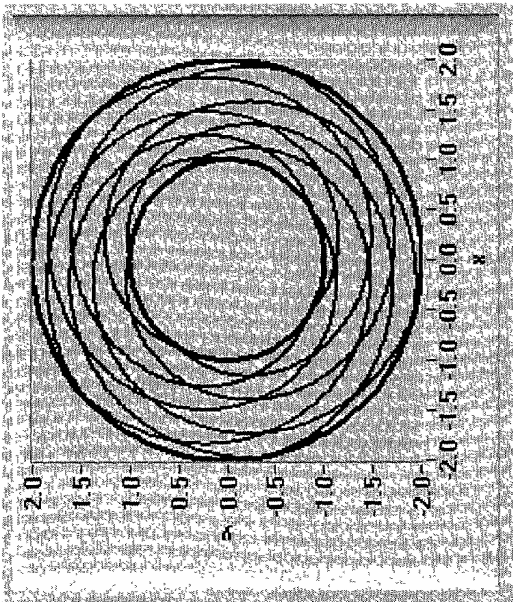
Figure 14

Low-discrepancy curve in a ring

Figure 15B
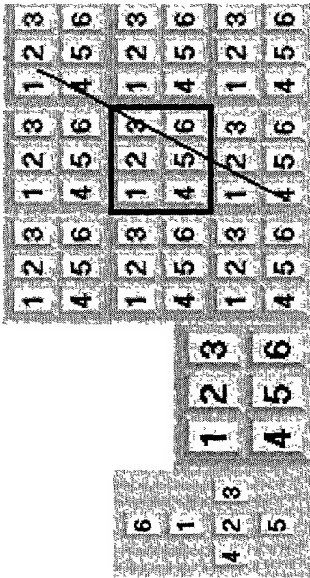


Tiling of the plane and relation to the surface of the unit cube

Figure 15A

Low-discrepancy curve filling the surface of a torus
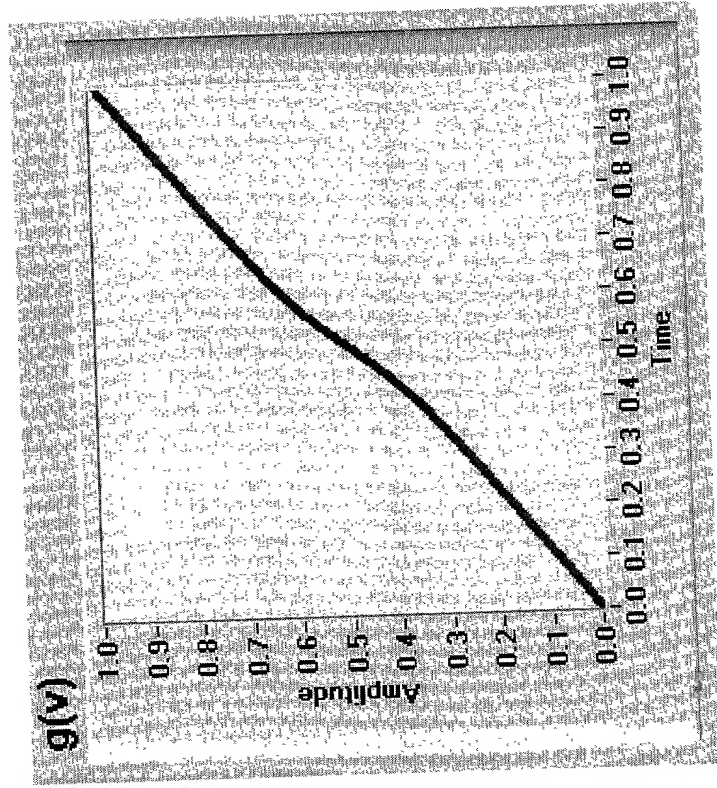
Figure 15D



Low Discrepancy Preserving Mapping Function
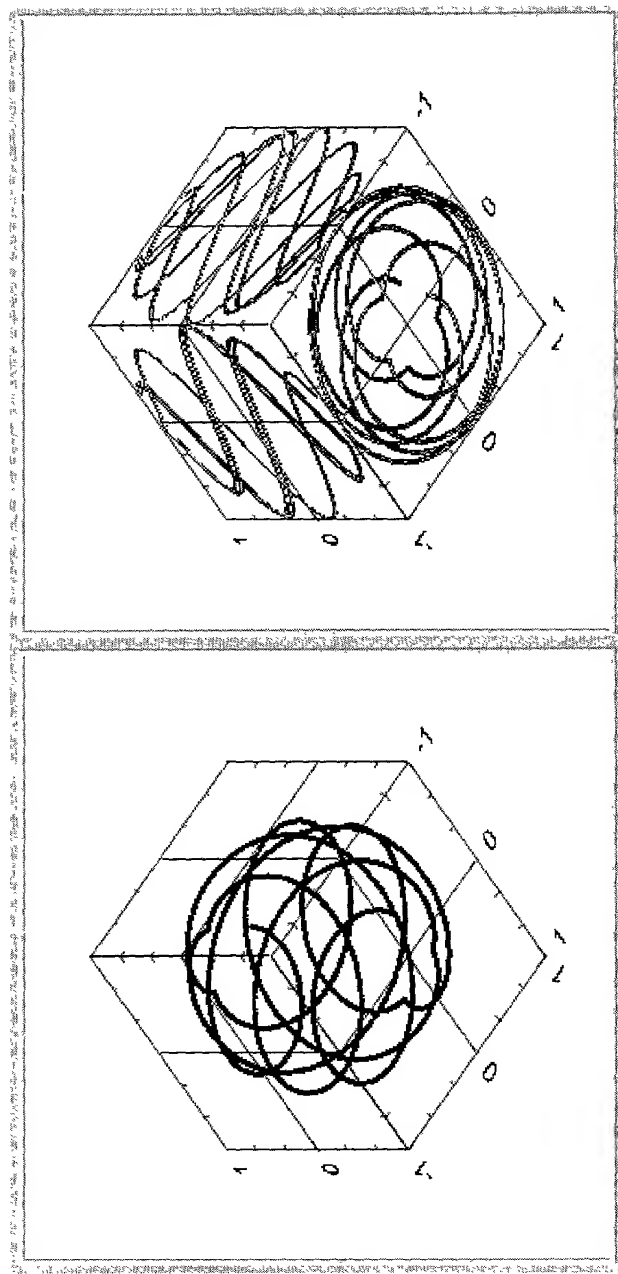
Figure 15C

Low-discrepancy curve on a sphere
(left) and projections (right)

Figure 16

Select a Parameterization of a Surface S Where the
Parameter Space is the Unit Square
1702

Select a LD Curve in the Parameter Space, i.e.,
Select a Suitable (alpha1, alpha2)
1704

Determine a Re-Parameterization of S Such That
the Ratio of Line and Area Elements of S, Based on
a Riemannian Metric, is Constant
1706

Map the LD Curve in the Unit Square onto the
Surface S Using the Re-Parameterization
1708

Output the Mapped LD Curve on S
1710

Figure 17

Surfaces can be scanned efficiently when the term low discrepancy sequence/ curve can be generalized, e.g. based on metrics on the surface.

Figure 18
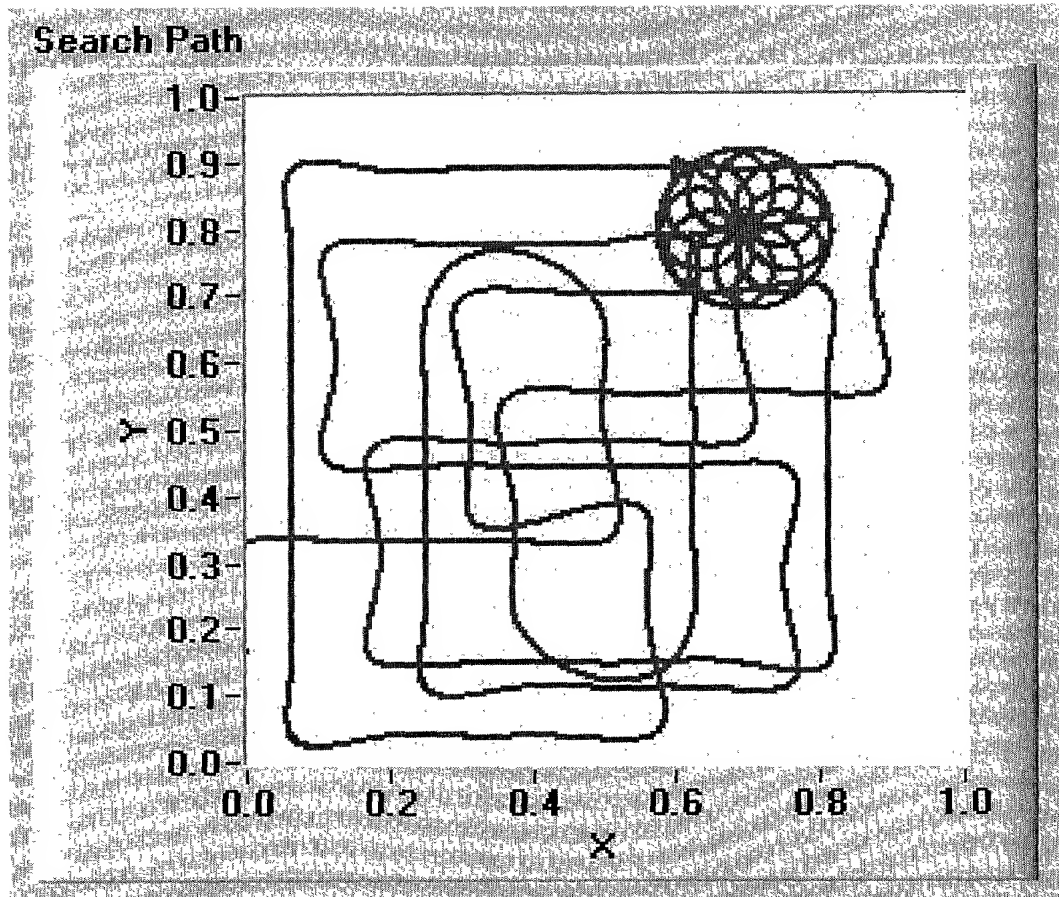
Splined Low Discrepancy Curve coarse search with refined final approach

Figure 19

**Intensity Field Distribution in Search Area**
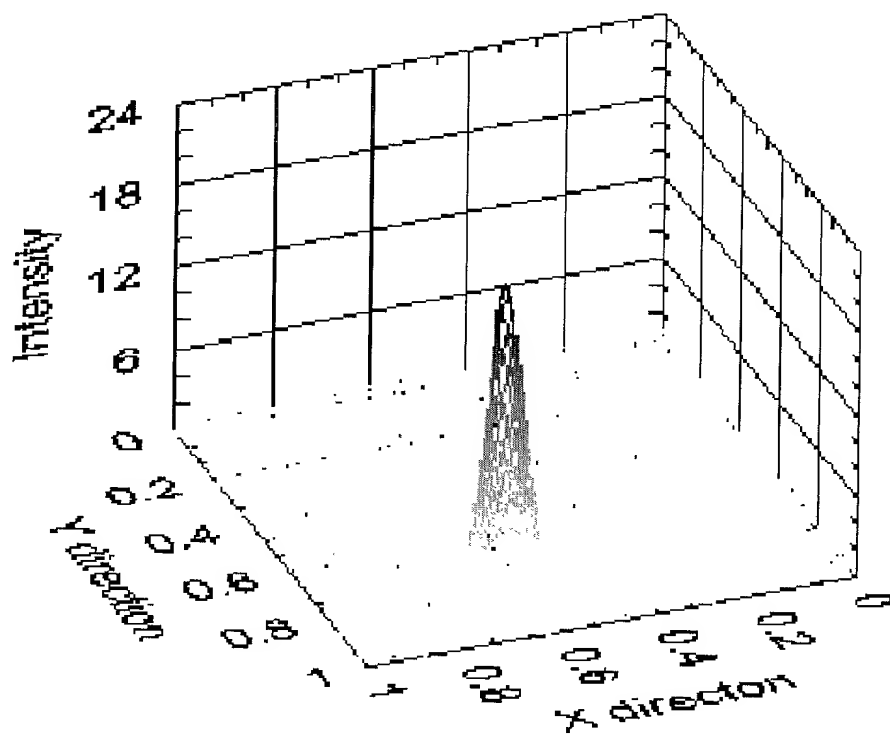


Beam intensity distribution in search area

Figure 20
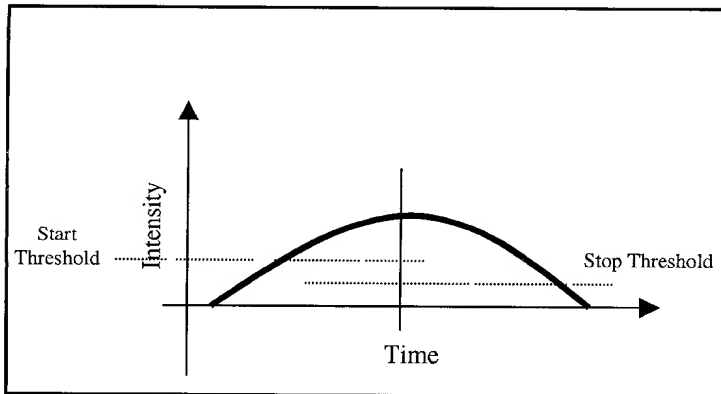
Location of the Peak
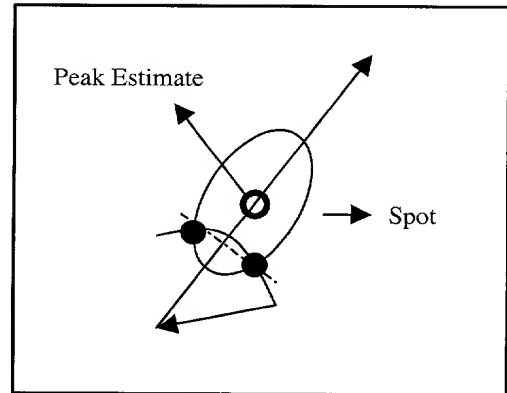
**Figure 21A**



Initial Final Approach Move

**Figure 21B**
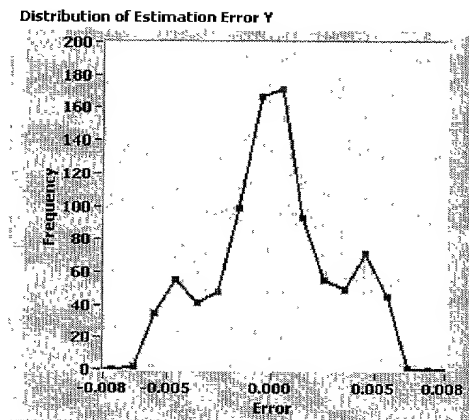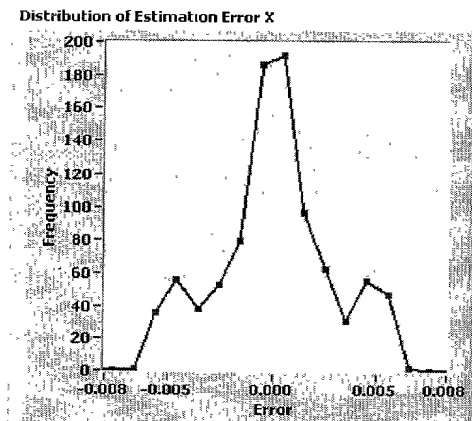


Error distribution of the estimated peak X coordinate error (left) and Y coordinate error (right)

**Figure 21C**

```
        ┌─────────┐
        │  Start  │
        └─────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│ Determine Characteristics of Region of Interest in the │
│                    Region                      │
│                     2202                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│   Calculate a Continuous Scan Trajectory Based On   │
│    the Determined Characteristics of the Region of   │
│                    Interest                    │
│                     2204                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│  Measure Region Along Continuous Scan Trajectory   │
│        to Generate a Sample Data Set           │
│                     2206                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│   Fit the Sample Data Set With a Parameterized    │
│        Surface Using A Model of the Region        │
│                     2208                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│   Calculate  Location of Point of Interest Based on  │
│             Parameterized Surface              │
│                     2210                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│    Measure  the Region at the Point of Interest to   │
│        Confirm the Calculated  Location          │
│                     2212                       │
└───────────────────────────────────────────────┘
             │
             ▼
┌───────────────────────────────────────────────┐
│  Generate Output Indicating the Determined Object   │
│                Characteristics                 │
│                     2214                       │
└───────────────────────────────────────────────┘
             │
             ▼
        ┌─────────┐
        │   End   │
        └─────────┘
```

Figure 22

```
                          ┌──────────┐
                          │  Start   │
                          └────┬─────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Scan Region to Determine An Entry Point and An │
        │       Exit Point of the Region of Interest       │
        │                     2302                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Determine First Local Point of Interest By Scanning │
        │  Along First Scan Line Between Entry and Exit Points  │
        │                     2304                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │  Calculate Second Scan Line Passing Through First │
        │  Local Point of Interest and Orthogonal to First Scan │
        │                     Line                         │
        │                     2306                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │   Scan Along Second Scan Line to Generate Second  │
        │            Scan Line Measurement Data             │
        │                     2308                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │   Determine Second Local Point of Interest Along   │
        │       Second Scan Line Based on Second Scan Line   │
        │                  Measurement Data                 │
        │                     2310                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
        ┌──────────────────────────────────────────────┐
        │    Determine Approximate Center of the Region of  │
        │  Interest Based on First and Second Points of Interest │
        │                     2312                         │
        └──────────────────────┬───────────────────────┘
                               │
                               ▼
                          ┌──────────┐
                          │   End    │
                          └──────────┘
```

Figure 23